**Unit 1: Number Sense &**

**The Arithmetic of Whole Numbers**

**Lecture Notes:**

|  |  |
| --- | --- |
| **Main Ideas** | **Details** |
| **Rounding** | 1. **You must know to what place value you’re rounding** 2. **Look at the digit to the right of that place value** 3. **If it’s 5 or above, round up** 4. **If it’s 4 or below, round down** 5. **Round the number above to the ten-thousands place. To the millions place?** |
| **Estimation** | * **Approximately how much is the solution?**  1. **Write each number in the problem with only one non-zero digit** 2. **Solve** |
| **Place Value** | * **Numeral=single counting number** * **Digit=numeral within a larger number that holds a place value** * **Place Value**   + **Standard form**   + **Expanded Form** * **Write the following number in expanded form: 1,354,294,466** * **What place value does the “2” hold? The “5”?** |
| **Basic Operations** | * **Addition +** * **Subtraction -** * **Multiplication ×** * **Division ÷** |
| **Symbols** | **< less than**  **> greater than**  **≤ less than or equal to**  **≥ greater than or equal to**  **≠ not equal to**  **= equal to**  **≅ approximately equal to**  **≈ approximately**   * **Create number sentences with symbols** |
| **Number Line** | **What is the distance the frog traveled in the first hop? The second? What number sentence can we create with this number line?** |
| **Addition** | * **Terms/addends=the numbers in the problem** * **Sum=the answer** * **Addition is commutative=the order of the terms doesn’t matter** |
| **Multiplication** | * **Multiplication is related to addition because it’s a faster way to perform additive operations** * **Factors=the numbers in the problem** * **Product=the answer** * **Prime numbers have only two factors: 1 and itself** * **Composite numbers have more than two factors** * **Any number can be broken down to a series of prime factors through prime factorization** * **Rule: Multiplying any number by 0=0** * **Notation** * **a x b** * **a \* b** * **a(b)** * **ab** * **a b** |
| **Subtraction** | * **Difference= the answer** * **Subtraction is the inverse (opposite) of addition** |
| **Division** | * **Division is related to subtraction because it’s a faster way to perform subtractive operations** * **Subtraction and division are NOT commutative!** * **Quotient: the answer** * **Dividend: the first number you say in a division equation** * **Divisor: the second number you say in a division equation** * **Ex: 21 ÷ 3 = 7 (21=dividend; 3=divisor) AKA 3∫21 (it’s backwards with the division bar)** * **Notation**   + **21 ÷ 3 = 7**   + **3∫21**   + **21/3**   + **21**   **3**   * **Rule: You cannot divide by zero! (when zero is the divisor) Division by zero is undefined.**    + **Example: 0 ÷ 9 makes sense because if there are 9 apples in total to eat, you don’t eat any. 9 ÷ 0 doesn’t make sense because there are 0 apples in total to eat, but you eat 9 (impossible)** * **Division is the inverse (opposite) of multiplication** |
| **Order of Operations** | * **Please Excuse (My Dear) (Aunt Sally)**   + **Parentheses**   + **Exponents and Radicals**   + **Multiplication & Division**   + **Addition & Subtraction** * **Example: 3 + (7-5)3-8÷(4-2)** |

### Properties of Real Numbers

### Fundamental Theorem of Arithmetic

Any integer greater than 1 can be written as a unique product of prime numbers.

### Property of Comparison

If *a* and *b* are real numbers, then one and only one of the following statements is true:

*a < b, a = b, or a > b.*

### Transitive Property of Order

If *a, b,* and *c* are real numbers, then the following statements are true:

If *a < b* and *b < c,* then *a < c.*

If *a > b* and *b > c,* then *a > c.*

### Property of Closure for Addition and Multiplication

If *a* and *b* are real numbers, then both of the following statements are true:

*a + b* represents a unique real number, and

*ab* represents a unique real number.

### Properties of Equality

If *a, b, c,* and *d* are real numbers, the following axioms are true.

|  |  |
| --- | --- |
| Name of Axiom | Statement |
| Reflexive | *a = a* |
| Symmetric | If *a = b,* then *b = a.* |
| Transitive | If *a = b* and *b = c,* then *a = c.* |
| Substitution | If *a = b* and *a + c = d,* then *b + c = d.*  If *a = b* and *ac = d*, then *bc = d.* |

### Commutative Property

If *a* and *b* are real numbers, then *a + b = b + a* and *ab = ba.*

### Associative Property

If *a*, *b*, and *c* are real numbers, then (*a* + *b*) + *c* = *a* + (*b* + *c*) and (*ab*)*c* = *a*(*bc*).

### Distributive Property

If *a*, *b*, and *c* are real numbers, then

*a*(*b* + *c*) = *ab* + *ac* and *a*(*b* - *c*) = *ab* - *ac*.

### Identity Elements

In the set of real numbers, the number 0 is the identity element for addition because *a +* 0 *= a.*

In the set of real numbers, the number 1 is the identity element for multiplication because 1 • *a = a.*

### Definition of Reciprocity

Two non-zero real numbers are considered to be reciprocals of one another if their product is 1:

* =* 1*.*

### Division by 0 Is Meaningless

In the real number system, division by 0 is meaningless.

## The Rules of Arithmetic | Problems

**Directions**: Each problem is an example of one of the basic rules of arithmetic. Write the name of the rule on the line.

1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 4 + 3 = 3 + 4
2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 4 • 5 + 4 • 3 = 4(5 + 3)
3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 2 + (3 + 4) = (2 + 3) + 4
4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 0 • 25 = 0
5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 12 • 1 = 12
6. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 1984 + 0 = 1984
7. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 4(6 • 3) = (4 • 6)3
8. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 7(*x* + *y*) = 7*x* + 7*y*
9. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (*p* + *q*) + *r* = *p* + (*q* + *r*)
10. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ *mn* = *nm*

**Directions**: Use the distributive property to rewrite these expressions without parentheses.

1. 4(*x* + 3) =
2. 6(*a* – 5) =
3. 12(*y* + 1) =
4. 9(*b* – 6) =
5. 16(*t* + 2) =
6. 21(*z* – 2) =
7. 8(*u* – 3) =
8. 7(*p* + 12) =

**Directions**:Use the distributive property to factor out common factors and rewrite these expressions with parentheses.

1. 10*x* + 90
2. 3*y* + 36
3. 4*z* – 8
4. *13c* + 39

## The Rules of Arithmetic | Answers

1. commutative property
2. distributive property
3. associative property
4. multiplication property of zero
5. multiplication property of one
6. addition property of zero
7. associative property
8. distributive property
9. associative property
10. commutative property
11. 4*x* + 12
12. 6*a* – 30
13. 12*y* + 12
14. 9*b* – 54
15. 16*t* + 32
16. 21*z* – 42
17. 8*u* – 24
18. 7*p* + 84
19. 10(*x* + 9)
20. 3(*y* + 12)
21. 4(*z* – 2)
22. 13(*c* + 3)